The payment of wages[1]

Alberto Benítez Sánchez[2]

Abstract

This article studies the effects on income distribution caused by changes in the fraction of wages paid at the start of a production program, when the remaining wage is paid at the end. Among other results, it is found that this fraction permits the definition of certain restrictions affecting the form of the wage-profit curve as well as other constraints influencing Wicksell effects on prices.

1. Introduction

In this article, I study some restrictions due to the schedule for the payment of labor that affect variables related to income distribution. Normally, changes in this schedule induce modifications to the production system via variations of aggregate demand in the different markets, while a constant production program permits the isolation of the effects of these changes over the variables being investigated. With the intention to preserve the last condition, I consider an equilibrium situation in which all the markets are cleared and all the industrial branches obtain the same rate of profit. In this framework, I analyze the relations between the rate of profit, income distribution and the schedule for the payment of salaries that are compatible with a given production program.

As is frequently the case when studying the interdependence between prices and production costs, I consider a linear model of single product industries with no fixed capital similar to those presented in Hawking (1948), Dorfman, Samuelson and Solow (1958), Sraffa (1960),

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[1] Translation of Benítez (2009).

[2] Economic Department, Universidad Autónoma Metropolitana (Iztapalapa).

Leontief (1966), Morishima (1973), Broome (1983) and Roemer (1983), which have also been investigated by several other authors. A common feature of these models is that salaries are paid at a single date either starting or ending the production process but, with the intention of exploring the relations mentioned above, I assume that a fraction t $(0 \leq t \leq 1)$ of them is paid when production begins and the rest when it finishes. Labor is the only input that I take as being employed forcefully in every industry and it is not necessarily homogeneous in kind. A brief summary of the model is offered in the second section and in the third one I present some comments about its features that are relevant to this research. The main results are presented in the following order.

In a theorem introduced in the second section and proved in Appendix A, I establish that the system of relative prices corresponding to any given rate of profit (r) as well as the maximum level of this variable (R) is independent of t. Nevertheless, as shown in the fourth section, for each $r \in \,]0,R[$, the wage unit measured with the net product (w) is a monotonous decreasing function of t. This implies that for each $r \in \,]0,R[$ and for each $w \in \,]0,1[$, there is respectively a corresponding interval of possible values of w and of r determined by the different levels of t.

Moreover, there is a particular restriction –pointed out in the fifth section– limiting the magnitude of each one of the variables w and r that is independent of the technique used in the system. It is determined respectively by the values of r and t (for w) and of w and t (for r). From this, I derive some consequences affecting the form of the wage–profit curve, proving that it may be a straight line only if $R \leq 1/t$, strictly concave only if $R < 1/t$ and, if it has a single profile all over the interval $[0,R]$, when $R > 1/t$ it can only be strictly convex. I also develop a formula to estimate the amount of capital advanced (K) measured with the net income given the values of r and t.

Harcourt (1972, pp. 39–46) explains that a change in capital caused by a reduction in the rate of profit taking place while the technique remains unaltered is known in the modern literature as a price Wicksell effect (PWE); it is either negative, neutral or positive if capital respectively diminishes, remains constant or increases. With regard to closely related matters, Broome (1983, pp. vi–ix, 56) sustains that when salaries are advanced it is more appropriate to consider them as being a part of capital. I adopt these definitions and in the sixth section I identify three conditions related to the form of the wage–profit curve that are necessary and sufficient for $K(r)$ to be a monotonous function respectively constant, increasing and decreasing. I also show that when the PWE is of a single type all over the interval $[0,R[$ in a given production system, it can be neutral only if $R \leq 1/t$, negative only if $R < t$ and if $R > 1/t$ it can only be positive. However, normally the PWE is not of a single type. Considering the general case, I present a formula that depends only on t and R permitting the calculation, under a particular assumption, of an upper bound to the proportion between the non-necessarily positive and the positive PWE if $R > 1/t$.

It is worth noticing that wt increases monotonously when r diminishes, while at the same time the value of the physical means of production measured with the net product may grow, decrease or remain constant. For this reason, it seems convenient in the present context to refer to this value as the capital stock (KS). Therefore, I will distinguish a PWE from the consequence of a reduction in the rate of profit on the capital stock; they will be the same phenomenon only if $t = 0$. As proved in the same section, this consequence is not subject to the constraints exposed above.[3] In the last section, I present some general comments.

2. The model

The model represents a productive system integrated by n industrial branches, each one of them producing a particular type of good labeled by an index i or j, so that $i, j = 1,2,\ldots,n$. I will refer to a set $\{j1,j2,\ldots jd,\ldots jD\}$ as a D set if it contains D different goods.[4] All the production processes are simultaneous and of equal duration, the quantities of each good are measured with the amount produced of the corresponding good and the quantities of salaries with the amount of

[3] Alternatively, the concept of a PWE may be reserved for changes in the capital stock, introducing another term to designate changes in the capital advanced. Also, a distinction could be made between two types of PWE affecting respectively capital advanced and capital stock. Apart from being endorsed indirectly by Broome's arguments already mentioned, the option chosen here seems to be the simplest one.

[4] In order to simplify, I will also refer to the indexes as goods.

salaries paid. For each pair (i,j) of indexes, a_{ij} and l_j represent respectively the quantities of i and of salaries consumed in the j industry during the period considered to produce one unit of j; they are non-negative numbers verifying for each j that $l_j > 0$ while $a_{ij} > 0$ at least for one i.[5] A good i produces directly a good j (not necessarily different) if $a_{ij} > 0$ and indirectly if there is a D set containing neither i nor j and verifying $a_{i,j1}a_{j1,j2}a_{j2,j3}\ldots a_{jD,j} > 0$.

For each j, the price of good j in units of salary is p_j and r is the rate of profit of the period. Given the fact that $1 - t$ is the fraction of the wage paid at the end of production, the cost of labor in each branch j is $l_j t(1 + r) + l_j(1 - t) = l_j(1 + tr)$. In these conditions, if the rate of profit is the same in every branch, the prices and costs of production are related by the following system of equations:

$$\textstyle\sum_i a_{ij}p_i(1 + r) + l_j(1 + tr) = p_j \qquad j = 1,2,\ldots,n \qquad [1]$$

I will say that [1] is viable if in every D set the sum of the quantities of each good belonging to D that are consumed directly in the production of the goods of D is not greater than 1 and is less than 1 for at least one of the goods. Consequently, every D set verifies that $\sum_d a_{i,jd} \leq 1$ for each $i \in$ D and $\sum_d a_{i,jd} < 1$ for at least one $i \in$ D.

I assume that every economy considered in this work is viable, which, together with the other assumptions already made permit us to verify the following propositions for every t.

Theorem 1. There is an interval $[0,R[$ such that: a) R is independent of t and $0 < R < +\infty$, b) for each $r \in [0,R[$, the solution of [1] is unique and strictly positive, c) $p_j(r)$ is a monotonous increasing function for every j, d) at least one price tends to infinity when r tends to R, e) for each $r \in [0,R[$, the quotient $p_i(r)/p_j(r)$ is independent of t $\forall$ (i,j).

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[5] If the assumption is made that each quantity of salaries pays an equal quantity of labor, l_j may also be interpreted as the quantity of labor consumed in the *j-th* industry. However, this is not required for the purposes of the article. Some references concerning the use of the wage unit in the economic literature are presented in Kurz and Salvadori (1995, p. 116)

Proof. See Appendix A.

There are similar results in the books cited above by Broome, Kurz and Salvadori, Morishima, Roemer, Sraffa and in some other works. Nevertheless, due to the peculiarities of system [1] indicated in the introduction and also as a consequence of the viability concept adopted here, I prove Theorem 1 following procedures somewhat different from those already published.

For each i, $c_i = 1 - \sum_j a_{ij}$ represents the quantity of good i produced as surplus over the amount of the same good consumed as a means of production, as [1] is viable $c_i \geq 0\ \forall\ i$ and $c_i > 0$ for at least one i. Summing up the n equations of [1], we obtain $\sum_j \sum_i a_{ij} p_i (1 + r) + \sum_j l_j (1 + tr) = \sum_j p_j$. Substituting $\sum_j l_j$ and $\sum_j p_j$ with their respective equivalents 1 and $\sum_j \sum_i a_{ij} p_i + \sum_j c_j p_j$ in the previous equation yields $\sum_j \sum_i a_{ij} p_i (1 + r) + (1 + tr) = \sum_j \sum_i a_{ij} p_i + \sum_j c_j p_j$ so that:

$$\sum_j \sum_i a_{ij} p_i r + (1 + tr) = \sum_j c_j p_j \qquad [2]$$

The first term of the left side of this equation is the amount of profits obtained with the means of production and the second one that of wages together with the profits corresponding to the wages advanced. As the value of the collection of goods at the right side is equal to the net income of the society, I will refer to this collection as the real income.[6]

3. Wages and labor's cost

Measured with the real income, the capital stock and the value of the wage unit are determined respectively by the following equations:

$$\text{a) } KS = \sum_i \sum_j a_{ij} p_i / \sum_j c_j p_j \qquad \text{and} \qquad \text{b) } w = 1/\sum_j c_j p_j \qquad [3]$$

The wage, represented by the second variable, is the part of the net income corresponding to labor while wt and $w(1 - t)$ are respectively the fractions paid at the beginning and the end of production. As $1 - w$

[6] Pasinetti (1977, p. 134) points out that when wages are paid at the beginning of production they are not included in the classical notion of net product. Nevertheless, independently of the schedule for the payment of wages, the value of the real income is equal to the net income.

corresponds to profits, [3.b] indicates the distribution of that aggregate between wages and profits. For this reason, when $t = 1$, the graph of $w(r)$ is the wage–profit curve of the economy as defined by Broome (1983, p. 14) or the factor–price frontier according to Morishima (1973, p. 56), who follows Samuelson (1962) in this point. Consequently, the graph of $w(r)$ may be identified with these expressions for every t.

Apart from w, the workers receive an additional benefit due to the fact that some salaries are advanced. It may be evaluated as equal to the profit that can be obtained if wt is employed as capital during the production period. Although the benefit is real, I find it appropriate to say that this quantity represents a virtual income insofar as it is not included in the value of the real income. As pointed out in the previous comments to equation [3.b], the last aggregate is equal to the sum of the actual wages plus profits; consequently, only that part of wt destined by the workers to obtain profits could give them access to an extra share of it. For instance, let us suppose that the workers do not require wt at the beginning of production while at the same time the enterprises are not in possession of this amount. Then, the first can accept postponing the collection of wt until the end of production in exchange for compensation determined by the rate of profit. In this case, the virtual income is reduced to zero because the workers receive a profit of wtr.[7] If, in a different situation, the workers consume the wage advanced, the virtual income remains equal to wtr, representing the opportunity cost that they pay for consuming that part of their earnings instead of using it as capital. It is important to remark that the distribution of income between wages and profits is independent of the amount of profits earned by the workers.

The sum of the real and the virtual parts of the workers' income is equal to the fraction of the net product represented by the salaries actualized at the end of production (w), determined by the product $w(1 + tr)$. I will refer to it as the actualized wage for simplification. Measuring prices with the real product, system [1] can be written as

[7] From the point of view of the workers, the profit mentioned in this example may be considered more properly as interest on savings but I want to underline that, as the wages are unchanged, the part of the real income thus obtained corresponds to profits, an aspect of interest stressed by Marx (1967, Vol. III, p. 370) who wrote: "Interest, as we have seen in the two preceding chapters, appears originally, is originally and remains in fact merely a portion of the profit...".

$$\Sigma_i a_{ij} p_i (1 + r) + w l_j (1 + tr) = p_j \qquad j = 1,2,\ldots,n \qquad [4]$$

It is easy to note that, for every given $r \in [0,R[$, the cost of production does not change in any industry j if its wage cost $wl_j(1 + tr)$ is substituted in [4] for the actualized wage equivalent $\text{w}l_j$. Therefore, the appropriate uses of both concepts are equivalent with regard to the system of relative prices corresponding to each $r \in [0,R[$. Nevertheless, $1 - \text{w}(r)$ is equal to the profit corresponding to the capital stock which is less than the capital's share of income and, consequently, $\text{w}(r)$ does not indicate the distribution of income between wages and profits in the model. For this reason, although its meaning will be further explored in Lemma 3, in the rest of the article I will consider only the wage.

From the point of view of the enterprises, the difference between w and w is the same as that existing between the quantity of salaries paid and the cost of labor at the end of production, which are respectively the quantity paid directly to the workers and the cost of this amount actualized at the end of production. The last quantity always counts as a part of the prices and hence of the net income while this does not occur with the virtual part of the workers income.

Equations [3.a] and [3.b] allow me to write [2] as the first of the following equations:

$$\text{a) } KSr + w(1 + tr) = 1 \text{ and b) } KS + wt = (1 - w)/r \ \forall\, r \in]0,R[\qquad [5]$$

The second equation is inferred from the first one; it establishes a relation between capital ($K = KS + wt$), the rate of profit and the distribution of income that will be studied in the next sections. However, it is convenient to remember here the following well-known properties of w as a function of r: I) [5.a] implies that $w = 1$ when $r = 0$, II) [3.b] and c) of Theorem 1 imply that w is a monotonous decreasing function of r and III) given the viability of [1], each good that is not produced as surplus produces at least one good present in the real income;[8] this

[8] Indeed, let D be the set of all the goods produced either directly or indirectly by i; as (1) is viable at least one good j belonging to D is consumed in the production of the goods of D in a quantity smaller than 1. The surplus is not consumed in the production of any good so that it belongs to the real income.

result and d) of Theorem 1 imply that when r tends to R at least one price in the denominator of [3.b] tends to infinity, making w tend to 0 at the same time. I will represent with W the set of all the continuous functions f: $[0,R[\rightarrow]0,1]$, where $R > 0$, whose graph verifies the properties indicated in I), II) and III). It is convenient to remark that all the $w(r)$ functions are determined by at least one system of type (1) and, as I consider only viable systems, they all belong to W, but there may be functions belonging to W for which there is no such system.

3. Prices and income distribution

Although relative prices normally change when r changes, as shown in Sraffa (1960, pp. 37–38), proposition e) of Theorem 1 indicates that, for any given $r \in [0,R]$, the relative prices are the same if the wage is paid entirely at the beginning or at the end of the production process and also if one part of it is paid at the first date and the rest at the second one.[9] By a procedure similar to the one followed in the proof of e), the same can be established for any schedule of payment provided that the workers receive the same fraction of their salary on each payment date in every industry. Nevertheless, this will not be the case if the industries follow different payment schedules. For instance, if starting in a situation where salaries are paid at the end of production the first industry decides alone to make this payment at the beginning, its cost will rise for any $r > 0$, but this will not affect those industries where the first good is used neither directly nor indirectly. As I will now show, these results have some consequences upon the relation between prices and income distribution.

The next two equations follow respectively from [5.a] and [5.b]:

$$\text{a) } w = (1 - KSr)/(1 + tr) \quad \text{and} \quad \text{b) } r = (1 - w)/(KS + wt) \qquad [6]$$

As may be appreciated in [3.a], KS depends solely on relatiO ve prices and for this reason e) of Theorem 1 implies that, for any given $r \in [0,R[$, the magnitude of KS will be the same independently of t. Consequently, according to [6.a], for any given $r \in]0,R[$, w is a

[9] Bidard (2004, p. 39) shows that, if r is constant, the relative prices are independent of the payment of the whole wage *ante* or *post factum*.

monotonous decreasing function of t that reaches its minimum value when the whole wage is advanced and its maximum when it is paid at the end of production while, according to [6.b], for any $w \in]0,1[$, r is a monotonous increasing function of t reaching its maximum and minimum values respectively when the whole wage is paid at the beginning and the end of production. As these equations show, the importance of t comes from the difference between the quantity of salaries paid and the cost of labor already pointed out in the previous section. Both quantities are equal only if $t = 0$ or if $r = 0$, otherwise the cost of labor includes a part of profits so that it is greater than w. Figure 1 illustrates these results, presenting the graphs of two $w(r)$ functions, w_0 and w_1, that correspond respectively to $t = 0$ and $t = 1$ in a system producing 1 unit of a certain good with $1/2$ a unit of the same good and 1 unit of wage.

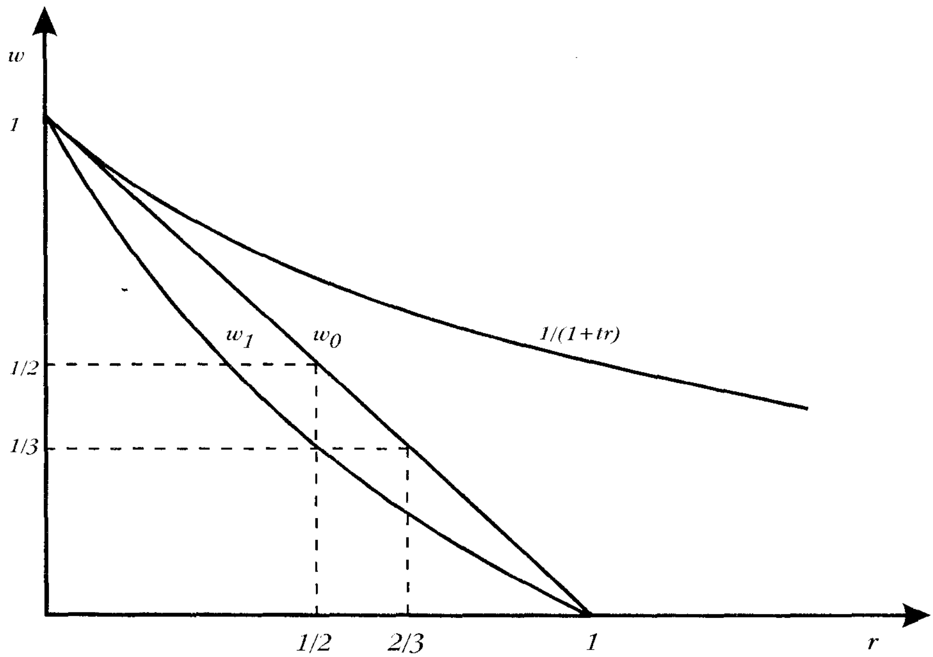

Figure 1. The graphs of $w_1(r)$, $w_0(r)$ and that of $1/(1 + tr)$ when $t = 1$.

The functions can be obtained by substituting in [6.a] KS for 1 in both cases, t for 0 in the first case and for 1 in the second case. When $r = 1/2$, w adopts each value of the interval $[1/3, 1/2]$ as t diminishes from 1 to 0. In the same figure, it can be appreciated that, for any given value of

$w \in]0,1[$, there will be an interval of possible values of the rate of profit and also that this rate may adopt any value pertaining to it depending on the particular schedule followed in the payment of wages. For instance, if $w = 1/3$, the rate of profit adopts each value of the interval $[1/2,2/3]$ as t diminishes from 1 to zero. These results mean that the distribution of income between wages and profits does not determine the rate of profit (and consequently the relative prices) nor vice versa, but each one of these variables determines the limits of the interval where the other one can adopt its value depending on the schedule for the payment of wages. Alternatively, it may be said that r determines w and vice versa on the condition that not only the production program but also t is given.

4. The form of the wage–profit curve

As $KS > 0$, equations [6.a] and [6.b] imply respectively the following inequalities, valid for every $r \in]0,R[$:

$$\text{a) } w < 1/(1 + tr) \qquad \text{and} \qquad \text{b) } r < (1 - w)/wt \qquad [7]$$

Each one of them imposes a restriction on the variable singled out that depends solely on the value of the other two variables. For this reason, the restriction on each variable is independent of the technique and is the same for all the systems of type [1] that share the same values in the right side of, respectively, [7.a] and [7.b]. It should be noted that, when t tends to 0, the limit of $1/(1 + tr)$ is 1 and that of $(1 - w)/wt$ is $+ \infty$. Therefore, each inequality establishes an effective restriction on the corresponding variable only if $t > 0$ but not when the wages are paid entirely at the end of production.

For $r \geq 0$, $1/(1 + tr)$ is strictly convex and a monotonous decreasing function of t and r. Its value is 1 when $r = 0$ and tends to zero when r tends to infinity, as is shown in Figure 1. The height of this curve increases when t diminishes, so that it tends to identify itself with the horizontal line of height 1 when t tends to 0.

Given a function f belonging to W and an r_b in the corresponding interval $]0,R]$, let Sr_b be the function determining for each r the height of the straight line that passes through the points $(0,1)$ and $(r_b, f(r_b))$. Then,

$$Sr_b(r) = 1 - r[1 - f(r_b)]/r_b \quad \forall\, r \qquad [8]$$

It should be mentioned that, if f is a $w(r)$ function, the absolute value of the slope of this line (which is the quantity multiplied by $-r$ in the equation) is equal to K when $r = r_b$, according to [5.b]. In a case particularly important for this analysis, when $r_b = R$, SR can be established by substituting in [8] $f(r_b)$ for 0 and r_b for R. After simplifying, we obtain $SR(r) = 1 - r/R \;\forall\, r$.

I will consider 4 subsets of W labeled W_1 to W_4 that are now to be described. The first one is integrated by all the functions whose graph is a straight segment. Therefore, among the $w(r)$ functions, it contains only those verifying

$$w(r) = 1 - r/R \;\forall\, r \in [0,R] \qquad [9]$$

W_2 includes the functions that lie entirely either above or below SR, except for their extreme points. Among the $w(r)$ functions, it contains only those satisfying one of the following inequalities:

$$\text{a) } w(r) > 1 - r/R \;\forall\, r \in\,]0,R[\text{ and b) } w(r) < 1 - r/R \;\forall\, r \in\,]0,R[\qquad [10]$$

W_3 is integrated by two types of functions that I define as above or below Sr if, given any $r_b \in\,]0,R]$, the corresponding level of the function is respectively above or below Sr_b for every $r \in\,]0,r_b[$. Finally, W_4 contains two types of well-known functions defined as strictly concave or strictly convex if, given any pair of successive values $(r_a,r_b) \in [0,R]$ for every $r \in\,]r_a,r_b[$, the corresponding level of the function is respectively above or below the straight line determined by the points $(r_a,f(r_a))$ and $(r_b,f(r_b))$. I will say that each one of the functions pertaining to the sets W_1 and W_4 possesses a single profile.[10]

[10] There may be concave and convex functions in W_2 that do not belong to W_3 and functions belonging to W_3 that are neither concave nor convex. Examples of the first two cases are the functions of W_2 that consist of two straight segments joining respectively above and below SR and of the last two cases the functions that belong to W_3 and are strictly concave or convex over $[0,R/2]$ and respectively strictly convex and concave over $[R/2,R]$. It is also worth noticing that $W_4 \subset W_3 \subset W_2$ but $W_2 \not\subset W_3 \not\subset W_1$. Indeed, $W_2 \not\subset W_3$, as can be verified with the first couple of examples just mentioned and the fact that $W_3 \not\subset W_4$ is verified with the last two examples.

The inequality [7.a] imposes some restrictions on the possible forms of the $w(r)$ functions that are established in the next two theorems. The first of them concerns W_1 and W_2 and the second one W_3 and W_4. Their proofs are based on the following proposition.

Lemma 1. Let $r_x > 1/t$; for each $r \in]0,r_x[$, the height of the straight segment that passes through the points $(0,1)$ and $(r_x,0)$, compared with $1/(1 + tr)$, is: a) greater if $0 < r < r_x - 1/t$, b) equal if $r = r_x - 1/t$ and c) less if $r > r_x - 1/t$.

Proof. For every $r_x > 0$, the equation of the straight line that contains the segment $[(0,1),(r_x,0)]$ is $1 - r/r_x = (r_x - r)/r_x$. Then, the difference between the two functions is equal to $(r_x - r)/r_x - 1/(1 + tr)$; by doing the subtraction we obtain $(r_x + r_x tr - r - tr^2 - r_x)/[r_x(1 + tr)]$, and by simplifying and dividing (as $r > 0$) the numerator by tr this quotient can be written as $[(r_x - 1/t - r)(tr)]/[r_x(1 + tr)]$. Consequently, the difference is bigger than, equal to and less than zero when respectively r is less than, equal to and greater than $r_x - 1/t$, finishing the proof.

This lemma is illustrated by Figure 2: when $r_x = 3/2$ and $t = 1$, the equation of the corresponding straight line is $1 - 2r/3$: its height is greater than, equal to and less than that of $1/(1 + tr)$ when r is respectively less than, equal to and greater than $1/2$.

Theorem 2. The function $w(r)$: a) can be either a straight line or above SR only if $R \leq 1/t$ and b) is below SR when $R > 1/t$, if it belongs to W_2.

Proof. According to b) of Lemma 1, if $R > 1/t$ there is an $r \in]0,R[$ such that $SR(r) = 1/(1 + tr)$; if either [9] or [10.a] holds, this r also satisfies $w(r) \geq 1/(1 + tr)$ in contradiction with [8], proving a). Consequently, if $R > 1/t$ and $w(r)$ belongs to W_2 only, [10.b] can be satisfied. For this reason, it is below SR, finishing the proof.

The graphs of functions w_0 and w_1 presented in Figure 1 are respectively a straight line and below SR. The graph of the $w(r)$ function corresponding to the following system:

$$(1/100)p_1(1 + r) + (99/100)(1 + r) = p_1 \quad [11]$$
$$(4/5)p_2(1 + r) + (1/100)(1 + r) = p_2$$

is above *SR*, as shown in Figure 2.

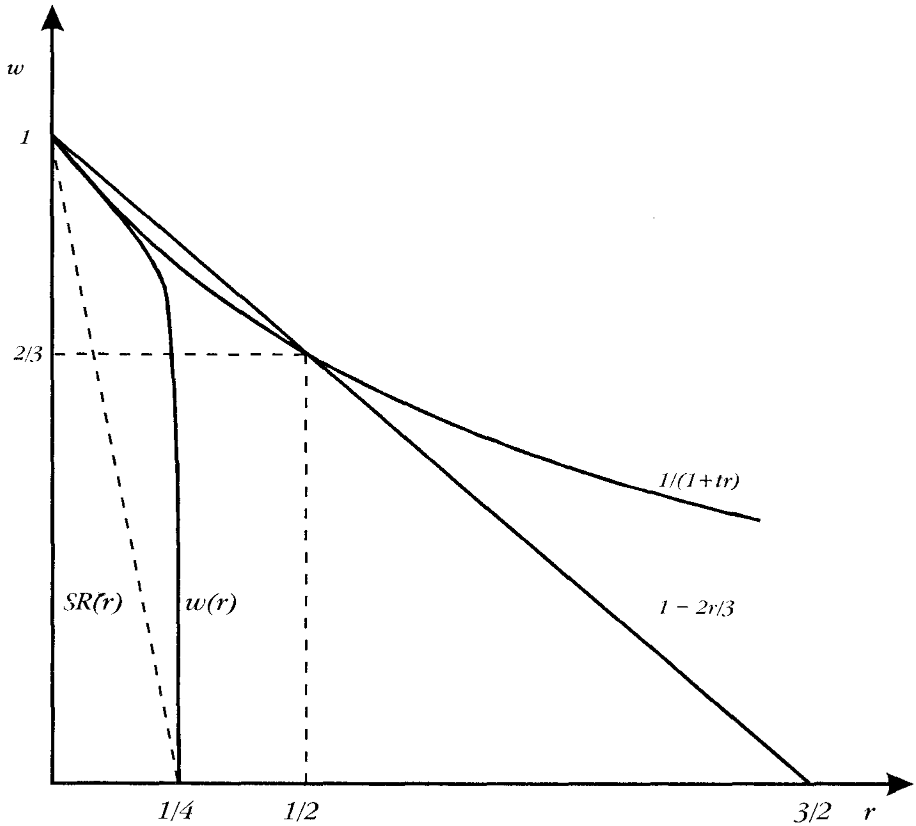

Figure 2. The graphs of $w(r)$ from system (11); $1/(1 + tr)$ when $t = 1$ and $1 - 2r/3$.

The income distribution in [11] is determined by:

$$w(r) = (1 - 4r)(99 - r)/(99 - 293.05r - 392.05r^2) \quad [12]$$

These conclusions are established in Appendix B, where system [11] is analyzed.

Theorem 3. If $w(r)$ belongs to W_3 or to W_4, it may be: a) strictly concave or above Sr only if $R < 1/t$ and b) only strictly convex or below Sr if $R > 1/t$.

Proof. The function $w(r)$ is strictly concave or above Sr only if [10.a] is satisfied, implying that $R < r_b/[1 - w(r_b)]$ for every $r_b \in]0,R[$. This permits us to verify that, for every $r_b \in]0,R[$, the r value (r_x) at the intersection of $Sr_b(r)$ with the horizontal axis is bigger than R. Indeed, substituting in [8] $Sr_b(r)$ for 0, f for w and r for r_x, we obtain $0 = 1 - r_x[1 - w(r_b)]/r_b$ and, consequently, $r_x = r_b/[1 - w(r_b)]$. Then, if $R \geq 1/t$, we have $r_x > 1/t$; this result and a) of Lemma 1 imply that there is at least one $r \in]0,r_b[$ such that $Sr_b(r) > 1/(1 + tr)$. The fact that $w(r)$ is either strictly concave or above Sr implies that this value of r also verifies the inequality $w(r) > Sr_b(r)$ and therefore $w(r) > 1/(1 + tr)$ in contradiction with [8], proving a). On the other hand, equations [9] and [10.a] can be satisfied only if $R \leq 1/t$ according to a) of Theorem 2. Then, if $R > 1/t$ and $w(r)$ belongs to W_3 or to W_4, only [10.b] can be verified; for this reason, $w(r)$ can only be either strictly convex or below Sr, finishing the proof.

The implications of [7.a] on the possible shapes of the $w(r)$ function established in this section have some consequences for the PWE that will be studied in the next one. Before that, I present here a formula to estimate capital given r and t.

Theorem 4. If $r \in]0,R]$, then

$$K(r) = (1 + 2tr)/[2r(1 + tr)] \pm 1/[2r(1 + tr)] \qquad [13]$$

Proof. From [3.b] and [7.a], we have $0 \leq w(r) < 1/(1 + tr)\ \forall\ r \in]0,R]$, so that $0 \geq -w(r) > -1/(1 + tr)$ and $1 \geq 1 - w(r) > 1 - 1/(1 + tr)$. As $1 - 1/(1 + tr) = r/(1/t + r)$, dividing by r the last inequalities yields $1/r \geq [1 - w(r)]/r > 1/(1/t + r)$ and, substituting the middle term of this expression for its equivalent according to [5.b], we finally have $(1/r) \geq K(r) > 1/(1/t + r)$. Therefore, $K(r)$ can be estimated as the average of the extreme values of the last interval with a maximum error equal to $1/2$ of the difference between these two values. Their sum is $(1 + 2tr)/[r(1 + tr)]$ and their difference is $1/[r(tr + 1)]$; dividing both formulas by 2, the proof is completed.

The maximum possible error as a fraction of the estimation is determined by $\{1/[2r(tr + 1)]\}/\{(1 + 2tr)/[2r(1 + tr)]\} = 1/(1 + 2tr)$. Consequently, this fraction is a monotonous decreasing function of r and t that tends to 1 when the product tr tends to 0 and to 0 as the product grows. For this reason, the formula is less good for small values of t and r, but its accuracy improves remarkably when the values of t and r increase. For instance, when $t = 1$ and r is successively equal to 2, 5, 10 and 20, the maximum error as a fraction of the estimation is respectively equal to $1/5$, $1/11$, $1/21$ and $1/41$.

The proportion between the investment and the real income, as the other relative prices, depends partly on the technology described by the technical coefficients and partly on the values of r and t. According to [13], the first part decreases as the product tr increases.

5. Price wicksell effects

Some relations between $K(r)$ and the form of the $w(r)$ function will be introduced now.

Theorem 5. The following are three pairs of equivalent propositions in the sense that each one of them implies the other one belonging to the same pair: a) $K(r)$ is monotonously increasing and $w(r)$ is above Sr, b) $K(r)$ is constant and $w(r)$ is a straight line and c) $K(r)$ is monotonously decreasing and $w(r)$ is below Sr.

Proof. The function $w(r)$ is above Sr if $\forall\ r_b \in\ ]0,R]$ and $r \in\ ]0,r_b[$:

$$w(r) > 1 - r[1 - w(r_b)]/r_b \Leftrightarrow w(r) - 1 > -r[1 - w(r_b)]/r_b$$

$\Leftrightarrow$

$$[w(r) - 1]/r > -[1 - w(r_b)]/r_b \Leftrightarrow [1 - w(r)]/r < [1 - w(r_b)]/r_b$$

As $r < r_b$, the last inequality implies, according to [5.b], that $K(r)$ is an increasing function $\forall\ r \in\ ]0,R[$. Consequently, if $K(0) \geq K(r_x)$ for an $r_x \in\ ]0,R]$, there is an $r_b \in\ ]0,r_x[$ such that $K(0) > K(r_b)$, but as $K(r)$ is continuous there is also in this case an $r \in\ ]0,r_b[$ for which $K(r) > K(r_b)$, contradicting the previous result and proving that $K(r)$ is an increasing function $\forall\ r \in [0,R[$. Or, if this is the case, the last of the four equivalent inequalities is verified $\forall\ r_b \in]0,R]$ and $r \in]0,r_b[$ so that $w(r)$ is above Sr according to the argument that starts with this inequality going backwards, proving a). To prove b) and c), it is enough to substitute in the preceding proof some easily identified words and symbols, apart from minor modifications in the case of b) when $r = 0$.[11]

These results permit us to establish, in the next proposition, a relation between changes in $K(r)$, the form of $w(r)$ and the restrictions to this form presented in the preceding section.

Theorem 6. If $K(r)$ is a monotonous function then it can be: a) increasing only if $R < 1/t$, b) constant only if $R \leq 1/t$ and c) only decreasing if $R > 1/t$.

Proof. a) of Theorem 5 and a) of Theorem 3 imply a), b) of Theorem 5 and a) of Theorem 2 imply b) while c) of Theorem 5 and b) of Theorem 3 imply c), ending the proof.

The conclusions just proved do not exclude that in some viable systems capital may grow and diminish consecutively when r increases. Nevertheless, given the constraint imposed by [7.a] for any t, when R increases the fraction of the interval $]0,R[$ over which the graph of $w(r)$ may be either concave or a straight line diminishes. A consequence of this is that, if a particular assumption presented below is adopted, the PWE will tend to be mostly positive, a result based in the following proposition.

Theorem 7. Given a pair $(r_x,r) \in [0,R]$ so that $r_x > r$ and $r_x > 1/t$, if the rate of profit descends from r_x to r, the PWE can be either neutral or negative only if $r > r_x - 1/t$.

Proof. For each $r_x \in]1/t,R]$, we have $w(r_x) \geq 0$, then $K(r_x) \leq 1/r_x$ according to [5.b]. On the other hand, as established in a) and b) of Lemma 1, if $r \leq r_x - 1/t$ then $1 - r/r_x \geq 1/(1 + tr)$. This inequality and [7.a]

[11] Harcourt (1972, pp. 39–43) shows that, if the graph of $w(r)$ is a strictly concave, straight or convex line, then $K(r)$ is a monotonous function respectively increasing, constant and decreasing. He also presents references to the original contributors to these results.

imply that $1 - r/r_x > w(r)$ if $r > 0$ so that $r/r_x - 1 < -w(r)$ and $r/r_x < 1 - w(r)$. As $r > 0$, dividing the last inequality by r yields $1/r_x < [1 - w(r)]/r$. Consequently, $1/r_x < K(r)$ for every $r \in]0, r_x - 1/t]$. Besides, $K(0) = KS(0) + w(0)t$; given the fact that $w(0) = 1$, it follows that $K(0) > t$ and, as $r_x > 1/t$, we obtain $1/r_x < t < K(0)$. Then, $1/r_x < K(r)$ for every $r \in [0, r_x - 1/t]$. This permits us to conclude that $K(r_x) < K(r)$ for every $r \in [0, r_x - 1/t]$ so that if $r \leq r_x - 1/t$, the PWE is positive, finishing the proof.

To each vector (r_x, r) verifying that $0 \leq r < r_x \leq R$ corresponds one particular PWE resulting as a consequence of a reduction on the rate of profit from r_x to r. Given a system of type [1], let $W(R)$ be the set of all the vectors that satisfy the condition indicated, which is equal to the triangular surface determined by the points $(0,0)$, $(R,0)$ and (R,R) except for the segment $[(0,0),(R,R)]$, as shown in Figure 3.

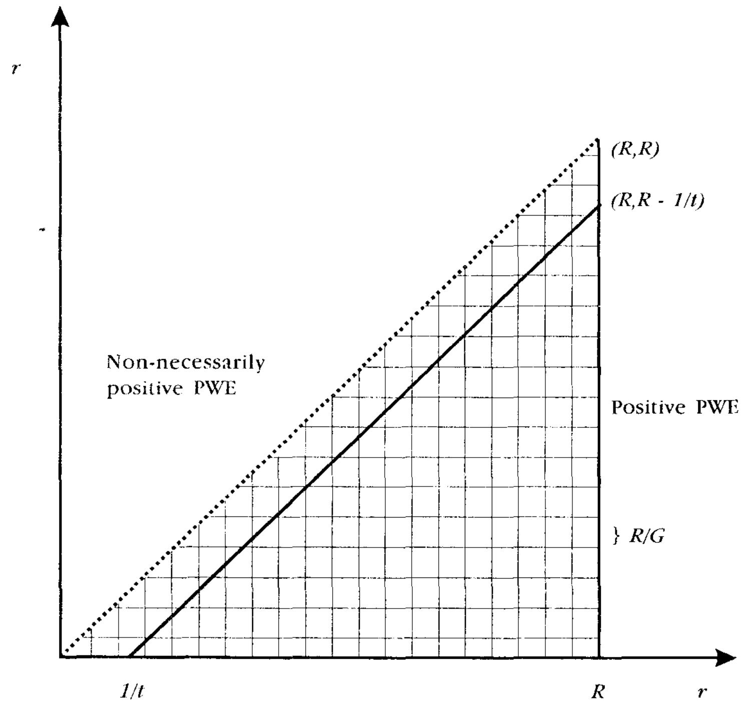

Figure 3. The $W(R)$ set when $R = 8$, $t = 1$ and $G = 20$.

For any $r_x > 0$, all the possible PWEs are associated with the points of the vertical segment $[(r_x,0),(r_x,r_x)[$. According to Theorem 7, if $r_x > 1/t$, the effects associated with the points in the segment $[(r_x,0),(r_x, r_x - 1/t)]$ are all positive while those that are not positive can be associated only with the points contained in $](r_x, r_x - 1/t),(r_x, r_x)[$, although the effect corresponding to any (or to all) of the points in this segment can also be positive.

In order to calculate an upper bound to the proportion between the non-necessarily positive and the positive PWE in a given system [1], it is convenient to consider a natural number $G > 2$ and to inscribe inside $W(R)$ $G - 1$ vertical and $G - 1$ horizontal segments separating each pair of parallel lines by a distance equal to R/G as shown in Figure 3. Let us suppose that G is so large that the difference between any two points contained inside the same square of side R/G is negligible for all the agents. Under this assumption, it is enough to consider, from their point of view, only those PWEs associated with the set $W(R) \cap \{G(R) \times G(R)\}$. Then, for any given G, the above-mentioned proportion is not greater than that existing between the number of elements belonging to two subsets of squares of side R/G contained in $W(R)$ and separated by the straight line of slope equal to 1 that passes through the point $(1/t,0)$: the first subset integrated by the squares whose right bottom corner is above the straight line and the second one by the squares whose corresponding corner is on and below the straight line. Because this quantity normally changes according to the magnitude of G, it is convenient to define the upper bound calculated here as the limit of the last quotient when G tends to $+\infty$. Consequently, the proportion between the two groups of PWE is not greater than the limit of the proportion between the two quantities just defined when R/G tends to zero. Thus, it is equal to the proportion between the areas of the corresponding sections of $W(R)$, which is determined by $[R^2/2 - (R - 1/t)^2/2]/[(R - 1/t)^2/2] = [R^2 - (R - 1/t)^2]/(R - 1/t)^2$; dividing each term of the right side of this equation by its denominator and simplifying, we arrive at the following conclusion.

Lemma 2. For any given t, the proportion between the non-necessarily positive and the positive PWE in a system of type (1) in which $R > 1/t$ is not greater than $[R/(R - 1/t)]^2 - 1$.

This lemma permits us to observe that, for any given t, the upper bound to the proportion between the two types of effects diminishes as R increases and that it tends to zero when R tends to $+\infty$. Also, for any given R, the upper bound tends to grow as t diminishes and it tends to $+\infty$ as t tends to $1/R$. Nevertheless, when R grows keeping t constant, the increase in the preponderance of the positive PWE is certain, something that does not necessarily occur with the non-positive PWE in the second situation. Although the subject is beyond the scope of this article, an interesting aspect of these results is that they may contribute to the study of this proportion, which is related to a theoretical debate exposed by Harcourt (1972), on empirical bases. Indeed, according to Lemma 3, the question about the proportion between the non-positive and the positive PWE in a productive system, either actually or as a tendency, can receive an approximate answer if a reliable estimation can be made for, respectively, the present and the future values of R and t (if $R > 1/t$).

Finally, as indicated in the introduction, the capital stock may not change in the same direction as the capital. This is illustrated by the system whose wage–profit curve is shown in Figure 1: KS is constant while capital diminishes monotonously when r grows if $t = 1$. Also, the variations in KS are not subject to the restrictions on capital derived from [7.a], as proved in the following proposition.

Lemma 3. The inequality [7.a] does not impose any restriction on the capital stock.

Proof. Solving [5.a] for KS, we obtain $KS = [1 - w(1 + tr)]/r$; substituting the actual for the actualized wage cost in this formula, we get $KS = [1 - \mathrm{w}(r)]/r$. Consequently, the variations of KS depend on the shape of the $\mathrm{w}(r)$ function, as may be inferred from Theorem 5. Nevertheless, whatever the form of this function or the value of R, for every $r \in]0,R[$, $\mathrm{w}(r) < 1$ so that $w(1 + tr) < 1$, which implies that in every case [7.a] is verified.

7. Conclusions

The preceding study shows that the schedule for the payment of wages constitutes a relevant variable affecting the interdependency between prices and income distribution in a productive system. This conclusion stands mainly on the results obtained concerning two themes: a) the effects of this schedule over the income distribution corresponding to each level of the rate of profit compatible with a given production program and b) the restrictions on the possible forms of the wage–profit curve determined by this schedule together with the maximum rate of profit, which permit us, among other things, to establish some general propositions relating to these two variables and the PWE. Generally speaking, these results are a consequence of the distinction studied in the third section between the wage and its actualized value which corresponds, from the point of view of the entrepreneur, to the distinction between labor's share in the real income and labor's cost.

The model that I introduced here covers a set of situations that is also partially considered by a large and diversified literature. Consequently, a proper comparison with these publications surpasses the scope of the present article, although the following observations – together with some comments already presented in the article – will help to place my contributions against the background of the previous studies.

To this end, it is convenient to distinguish three relevant positions in the economic literature regarding the schedule for the payment of wages. The first was advocated by Smith (1991) for whom this payment was generally done at the start of production so that wages are normally a part of capital, an assumption shared by other classical economist.[12] In contraposition, Marx (1991) sustains that the wages are always paid at the end of the period fixed by the labor contract, the workers giving credit to the enterprises. As the credit goes in the opposite sense when the wages are advanced, it may be concluded that the distinction between the wage and its actualized value was implicit in his analysis. Nevertheless,

[12] In page 32 he writes "In all arts, and manufactures the greater part of the workmen stand in need of a master to advance them the materials to their work, and their wages and maintenance till it be completed" and in page 133 "The far greater part of the capital of such master artificers, however, is circulated, either in the wages of their workmen, or in the prices of their materials, and repaid with a profit by the price of the work". (The first quote comes from Chapters VII Book One and the second one from Chapter I of Book Two). In the same sense Ricardo (1965, p. 53) wrote that "Capital is that part of the wealth of a country which is employed in production, and consist of food, clothing, tools, raw materials, machinery, etc., necessary to give effect to labor." (Chapter V).

he chooses to assume provisionally that $t = 1$ considering that this makes no alteration in the nature of the exchange of commodities, something confirmed by e) of Theorem 1 concerning relative prices.[13] Regarding this point, Negishi (1985, pp. 73-76) indicates that the assumption that wages are paid out of current, not past, output is proper of the post-Walrasian neoclassical school while the advancement of wages is more compatible with Marxist theory. The third position considers the payment of a part of wages at starting production and the rest when it finishes, which is the more appropriate interpretation according to Sraffa (1960). Nevertheless, this author decided to adopt 0 as the unique value of t in his model, probably assuming that this variable has no implications affecting the thesis presented in his book.[14]

From my point of view, the determination of the value of t that best represents the schedule for the payment of wages in a given economy requires empirical studies, a research that may be stimulated by the relevance of this variable, sufficiently argued in the article. Nevertheless, as the theoretical work on the subject can be carried out independently of the results of this research, I considered here all the possible values of t. On the other hand, it could be said that the approach that I followed is closer to the position preferred by Sraffa than to any of the two others. However, an important difference is that I do not assume that the fraction

[13] In page 174 he writes "In every country in which the capitalist mode of production reigns, it is the custom not to pay for labour power before it has been exercised for the period fixed by the contract, as, for example, the end of the week. In all cases, therefore, the use value of the labour power is advanced to the capitalist: the labourer allows the buyer to consume it before he receives payment of the price; he everywhere gives credit to the capitalist." In pages 174-175: "Nevertheless, whether money serves as a means of purchase or a as a means of payment, this makes no alteration in the nature of the exchange of commodities." and in the last page: "It will, therefore be useful, for a clear comprehension of the relation of the parties, to assume provisionally that the possessor of labour power, on the occasion of each sale, immediately receives the price stipulated to be paid for it." (Vol. I, Chapter VI).

[14] In page 9 he writes "In view of this double character of the wage it would be appropriate, when we come to consider the division of the surplus between the capitalist and workers, to separate the two component parts of the wage and regard only the 'surplus' part as variable; whereas the goods necessary for the subsistence of the workers would continue to appear, with the fuel, etc., among the means of production." And in page 10: "We shall, nevertheless, refrain in this book from tampering with the traditional wage concept and shall follow the usual practice of treating the whole wage as variable." He adds in the same page "In any case the discussion which follows can easily be adapted to the more appropriate, if unconventional, interpretation of the wage suggested above." (Chapter II, paragraph 6).

of the wages paid in advance covers the subsistence expenses of the workers, as Sraffa does.[15] Also, the claim that the results established in his work are compatible with the more appropriate interpretation of labor's payment can not be treated here because it requires a detailed evaluation of several issues.

Besides, the existence of a difference in the form of the wage–profit curve due to the payment of the whole wage at the beginning or at the end of production has already been noticed, for instance in Bidard (2004, p. 39), Pasinetti (1977, pp. 131–132) and Kurz and Salvadori (1995, p. 54). Nevertheless, –as far as I know– this is not the case with the relations between the different forms of the curve, the maximum rate of profit and the different schedules for the payment of wages.

Finally, Broome (1983, p. 56) states that "Wicksell effects are a nuisance in economics. The trouble is that they are unpredictable. A change of distribution will change the capital requirements of different products, but there is no simple rule that tells us in which direction or by how much". In this regard, I can say that Theorems 2 and 3 provide simple rules to reduce the possible forms of the wage–profit curve; Theorem 7 offers another rule that tells us in which direction the amount of capital changes and Theorem 4 gives us a formula permitting to estimate the magnitude of the change. Not one of the rules is general, but each one covers a number of cases that may be large, depending on the values of t and R. I should add that – to the best of my knowledge – Broome's comment is still valid considering the literature that was published after the appearance of his book.[16]

[15] About the subsistence wage, Roemer (1993, p. 33) pointed out that in advance capitalism "workers in fact do choose to consume different bundles and are not limited to subsistence in any meaningful sense".

[16] However, the results concerning the PWE are restricted by the particular definitions adopted in this article (see note 1). The most complete reviews of the related literature that I consulted were the aforementioned books by Bidard, by Harcourt and by Kurz and Salvadori.

APPENDIX A: Proof of Theorem 1

a) Let $A = [a_{ij}]$ be the $n \times n$ coefficients matrix. The fact that for each j $a_{ij} > 0$ for at least one i implies that in the canonical form of A there is at least one irreducible matrix, as shown in Lemma 1.1 by Seneta (1981, p 16). Therefore, the Frobenius root of A (λ_A) is greater than zero; if A is not indecomposable, λ_A is the greatest of the Frobenius roots of (possibly several) indecomposable matrices in the canonical form of A. Let A_c be one of these matrices and such that its Frobenius root is equal to λ_A. As (1) is viable, none of the sum columns of A_c is greater than 1 and at least one of them is less than 1; it follows that $\lambda_A < 1$. This is indicated in the first remark to Theorem 4.C.10 by Takayama (1987, p. 388). Defining

$$1/(1 + R) = \lambda_A \tag{A.1}$$

results in $R = (1 - \lambda_A)/\lambda_A$. Because $0 < \lambda_A < 1$ and does not depend on t, R satisfies a).

b) For each j, let p_j be the price of good j when $t = 1$. Introducing the $1 \times n$ matrices $\text{p} = [\text{p}_j]$ and $l = [l_j]$, when $t = 1$, system [1] can be represented by means of the equation $A\text{p}(1 + r) + l(1 + r) = \text{p}$. This equation can also be written as:

$$\{[1/(1 + r)]I - A\}\text{p} = l \qquad \text{[A.2]}$$

where I is the $n \times n$ identity matrix. If $r \in [0,R[$, then $1/(1 + r) > \lambda_A$ and for this reason there is a vector $\text{p} \geq 0$, $\text{p} \neq 0$ that satisfies [A.2], as indicated in proposition (II') of Theorem 4.D.2 by Takayama (1987, p. 392). The facts that $\text{p} \geq 0$ and $l_j > 0$ $\forall j$ imply that $\text{p} > 0$; this can be easily checked in any particular equation of [1]. The matrix $\{[1/(1 + r)]I - A\}$ is nonsingular according to (III') of the same Theorem, so that p is unique.

Defining $p = [p_j]$, system [1] may be represented by the equation $Ap(1 + r) + l(1 + tr) = p$. Multiplying its two sides by $(1 + r)/(1 + tr)$, we obtain $Ap[(1 + r)/(1 + tr)](1 + r) + l(1 + r) = p[(1 + r)/(1 + tr)]$. Let $\text{z}_j = p_j[(1 + r)/(1 + tr)]$ for each j and $\text{z} = [\text{z}_j]$, then (1) can be written in the form $A\text{z}(1 + r) + l(1 + r) = \text{z}$ and also as:

$$\{[1/(1 + r)]I - A\}z = l \qquad [A.3]$$

[A.2] and [A.3] imply that p = z. Accordingly, $p_j = p_j(1 + r)/(1 + tr)$ for each $r \in [0,R[$ and for each j. Therefore,

$$p_j = p_j(1 + tr)/(1 + r) \; \forall \; t \in]0,1] \qquad [A.4]$$

As b) is valid for $t = 1$, [A.4] proves its validity $\forall \; t \in [0,1[$.

c) Let $(r_1, r_2) \in [0,R[$, $r_1 < r_2$, $f_i = p_i(r_2)/p_i(r_1)$ for each i and $f_b = \min\{f_i \mid i = 1,2,\ldots, i\}$. When $r = r_2$ the *b-th* equation of (1) may be written as $\Sigma_i a_{ib}[f_i\, p_i(r_1)](1 + r_2) + l_b(1 + tr_2) = f_b p_b(r_1) \Rightarrow \Sigma_i a_{ib}[(f_i/f_b)p_i(r_1)](1 + r_2) + l_b(1 + tr_2)/f_b = p_b(r_1)$ substituting $p_b(r_1)$ we obtain $\Sigma_i a_{ib}[(f_i/f_b)p_i(r_1)](1 + r_2) + l_b(1 + tr_2)/f_b = \Sigma_i a_{ib} p_i(r_1)(1 + r_1) + l_b(1 + tr_1)$, the fact that $a_{ib} > 0$ for at least one i together with the previous definitions imply that $\Sigma_i a_{ib}[(f_i/f_b)p_i(r_1)](1 + r_2) > \Sigma_i a_{ib} p_i(r_1)(1 + r_1)$. Consequently, $l_b(1 + tr_2)/f_b < l_b(1 + tr_1)$ so that $f_b > 1$.

d) Let $S = \{r_n = R - R/n \mid n = 1,2,3,\ldots\}$; if no price tends to infinity when r tends to R ($r \in S$), there is a real number H so large that every price belongs to $]0,H]$ for each $r \in S$. In this case, d) implies that, when r tends to R, the sequence formed by each price j converges to a limit (p_j^L) contained in $]0,H]$; let $p^L = [p_j^L]$. Consider the two sequences formed associating with each $n \in N$ the same particular side of [A.2] when r tends to R ($r \in S$), with p and $1/(1 + r)$ adopting their respective values: both converge and, for every $n \in N$, their corresponding terms are equal. Consequently, their limits are also equal, so that $\{[1/(1 + R)]I - A\}p^L = l$. However, (VI') of Theorem 4.D.2 already cited states that this equation may be verified only if $[1/(1 + R)] > \lambda_A$, something that contradicts (A.1), proving e) when $t = 1$. Its validity $\forall \; t$ follows from this result and [A.4].

e) According to [A.4], $p_i/p_j = p_i[(1 + tr)/(1 + r)]/p_j[(1 + tr)/(1 + r)] = p_i/p_j$ for any pair (i,j), so that relative prices are independent of t, ending the proof of the theorem.

APPENDIX B: Analysis of System [11]

Solving [11] yields

$$p_1 = [99(1 + r)]/(99 - r)$$
$$p_2 = (1 + r)/[20(1 - 4r)] \qquad [A.5]$$

The net product consists of $99/100$ units of the first good and $1/5$ of the second one, so that its value in wage units is $.99p_1 + .2p_2$. By substituting the corresponding prices, we have $.99[99(1 + r)]/(99 - r) + .2(1 + r)/[20(1 - 4r)]$; substituting $.99(99)$ for 98.1 and summing up the two fractions, we obtain $[(98.1)(1 + r)(20)(1 - 4r) + .2(1 + r)(99 - r)]/[(20)(1 - 4r)(99 - r)]$; dividing by 20 the numerator and the denominator yields $[98.1(1 + r)(1 - 4r) + .01(1 + r)(99 - r)]/(1 - 4r)(99 - r)$. The numerator can be written as $(1 + r)[(98.01)(1 - 4r) + (.01)(99 - r)] = (1 + r)(98.01 - 392.04r + .99 - .01r) = (1 + r)(99 - 392.05r) = 99 - 392.05r + 99r - 392.05r^2 = 99 - 293.05r - 392.05r^2$. Therefore, the net product in wage units is equal to $(99 - 293.05r - 392.05r^2)/(1 - 4r)(99 - r)$; substituting the denominator at the right side of [6] for this formula, we obtain equation [12].

Equation [A.5] determines that $R = 1/4$, so that $SR(r) = 1 - 4r$. Given the fact that $w(r) - SR(r) > 0$ for every $r \in]0,1/4[$, it follows that $w(r)$ is above SR. Indeed, the inequality $w(r) - SR(r) > 0$ may be written as $(1 - 4r)Q - (1 - 4r) > 0$ where Q is the function multiplying $(1 - 4r)$ in equation [17]. Consequently, it is valid for every $r \in]0,1/4[$ if and only if $Q > 1 \Leftrightarrow [99 - r - (99 - 293.05r - 392.05r^2)] = 292.05r + 392.05r^2 > 0$, which is correct for every $r \in]0,1/4]$.

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